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Electromagnetic form factors of the $\Delta(1232)$ excitation

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Abstract. In the resonance region we have precisely determined the electromagnetic properties of the $\Delta(1232)$-resonance, in particular the $E2/M1$ ratio $R_{EM} = (-2.5\pm0.1)\%$. For pion electroproduction recent experimental data from Mainz, Bates, Bonn and JLab for Q^2 up to 4.0 $(\mathrm{GeV}/c)^2$ have been analyzed with the isobar model MAID. The extracted $E2/M1$ ratio shows, starting from a small and negative value at the real photon point, a clear tendency to cross zero, and becomes positive with increasing Q^2. This is a possible indication of a very slow approach toward the pQCD region. The $C2/M1$ ratio near the photon point is found as $R_{SM}(0) = (-6.5 \pm 0.5)\%$. At high Q^2, the absolute value of the ratio is strongly increasing, a further indication that pQCD is not yet reached. The electromagnetic-transition form factors of the $\Delta(1232)$ excitation are parameterized and fitted to the electroproduction data. This also shows a zero-crossing of the electric form factor G_E^* at $Q^2 = 3.6 \pm 0.5$ $(\mathrm{GeV}/c)^2$.

PACS. 13.40.Gp Electromagnetic form factors – 13.60.Le Meson production – 14.20.Gk Baryon resonances with $S = 0$ – 25.20.Lj Photoproduction reactions

1 Introduction

The determination of the quadrupole excitation strength $E_{1+}^{(3/2)}$ in the region of the $\Delta(1232)$-resonance has been the aim of considerable experimental and theoretical activities. Within the harmonic-oscillator quark model, the Δ and the nucleon are both members of the symmetrical 56-plet of $SU(6)$ with orbital momentum $L = 0$, positive parity and a Gaussian wave function in space. In this approximation the Δ may only be excited by a magnetic dipole transition $M_{1+}^{(3/2)}$ [1]. However, in analogy with the atomic hyperfine interaction or the forces between nucleons, also the interactions between the quarks contain a tensor component due to the exchange of gluons. This hyperfine interaction admixes higher states to the nucleon and Δ wave functions, in particular d-state components with $L = 2$, resulting in a small electric-quadrupole transition $E_{1+}^{(3/2)}$ between nucleon and Δ [2–4]. In addition, quadrupole transitions are possible by mesonic and gluonic exchange currents [5,6]. Therefore an accurate measurement of $E_{1+}^{(3/2)}$ is of great importance in testing the forces between the quarks and, quite generally, models of nucleons and isobars.

In a constituent quark model with two-body meson exchange currents, Buchmann and Henley [6] show that the nucleon-to-Δ transition quadrupole moment is proportional to the mean-squared radius of the neutron charge distribution. In a study with three different models, a quark model, a collective model and a pion cloud model, they find in addition that the quadrupole moment of the Δ itself is proportional to r_n^2, $Q_0^{\Delta^+} = (1\cdots5)r_n^2$ and that also the nucleon would have an intrinsic (unobservable) quadrupole moment of $Q_0^p = (-1\cdots-5)r_n^2$. Therefore, in these models the Δ-resonance would be an oblate object while the nucleon would be prolate. Since it is extremely difficult to measure the quadrupole moment of the Δ, the only observable quadrupole moment is the transition moment from nucleon to Δ, which can be extracted from the recent E/M analysis of Mainz or Brookhaven, with a mean value of $Q_{N\Delta} = -(0.095 \pm 0.01)\,\mathrm{fm}^2$ (see below).

The $E2/M1$ ratio, $R_{EM} = E_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ has been predicted to be in the range $-3\% \leq R_{EM} < 0\%$ in the framework of constituent quark [2,4,5,7], relativized quark [8,9] and chiral-bag models [10,11]. Considerably larger values have been obtained in Skyrme models [12]. In the small-ϵ expansion of chiral perturbation theory a detailed study of the Δ-nucleon transition form factors has also been performed [13]. However, at the photon point the transition moments are fixed by counter terms and the predictions are given as a low-Q^2 behaviour of the form factors. A comparison with the experimental form

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factors is however difficult as first of all the validity is limited to very small Q^2, and secondly the calculations yield complex form factors that do not correspond to the usual definition of a Breit-Wigner resonance but more to the definition of a resonance pole of the T-matrix. A first lattice QCD calculation resulted in a small value with large error bars ($-6\% \leq R_{EM} \leq 12\%$) [14]. And in a very recent calculation considering both quenched and unquenched 2-flavour theory [15], the R_{EM} and R_{SM} ratios have been calculated at low Q^2 in a remarkable precision. Both ratios are in the range of $(-2 \cdots -3)\%$ and are comparable with the experimental analysis. However, the connection of the calculations with the experimental data is not so evident. Clearly, the Δ-resonance is coupled to the pion-nucleon continuum and final-state interactions will lead to strong background terms seen in the experimental data, particularly in case of the small E_{1+} amplitude. The question of how to "correct" the experimental data to extract the properties of the resonance has been the topic of many theoretical investigations. Unfortunately, it turns out that the analysis of the small E_{1+} amplitude is quite sensitive to details of the models, *e.g.*, nonrelativistic *vs.* relativistic resonance denominators, constant or energy-dependent widths and masses of the resonance, sizes of the form factor included in the width, etc. In other words, by changing these definitions the meaning of resonance *vs.* background changes, too.

In order to study the Δ deformation, pion photoproduction on the proton has been measured by the LEGS Collaboration [16] at Brookhaven and by the A2 Collaboration [17] at Mainz using transversely polarized photons, *i.e.* by measuring the polarized photon asymmetry Σ. In particular, the cross-section $\mathrm{d}\sigma_\parallel$ for photon polarization in the reaction plane turns out to be very sensitive to the small E_{1+} amplitude. Assuming, for simplicity, that only the P-wave multipoles contribute, the differential cross-section is

$$\frac{\mathrm{d}\sigma_\parallel}{\mathrm{d}\Omega} = \frac{q}{k}(A_\parallel + B_\parallel \cos\Theta_\pi + C_\parallel \cos^2\Theta_\pi), \qquad (1)$$

where q and k are the pion and photon momenta and Θ_π is the pion emission angle in the c.m. frame. Neglecting the (small) contributions of the Roper multipole M_{1-}, one obtains [17]

$$C_\parallel / A_\parallel \approx 12 R_{EM}, \qquad (2)$$

because the isospin-$\frac{3}{2}$ amplitudes strongly dominate the cross-section for π^0 production.

In addition to $p\pi^0$, also the charged-pion channel $n\pi^+$ has been measured at MAMI and LEGS, allowing a complete isospin separation. With high precision, it was shown by Beck *et al.* [18] that the ratios determined from only π^0 production agree very well with the ratio of the isospin-(3/2) multipoles, obtained after a full isospin separation. This result will be very important for all forthcoming electroproduction experiments that are purely based on the $\pi^0 p$ channel.

In order to obtain the $C2/M1$ ratio and the form factors as functions of Q^2, pion electroproduction has been studied. At Mainz, Bonn, Bates and JLab different experiments have been performed, without polarization as well as single and double polarization.

Table 1. $E2/M1$ ratios for $Q^2 = 0$ from different analyses.

$R_{EM}(\%)$	Reference
-2.54 ± 0.10	Hanstein *et al.* [19]
$-2.5 \pm 0.1_{\text{stat.}} \pm 0.2_{\text{syst.}}$	Beck *et al.* [18]
$-3.07 \pm 0.26_{\text{stat.+syst.}} \pm 0.24_{\text{mod.}}$	Blanpied *et al.* [16]
-2.0 ± 0.2	Arndt *et al.* [20]
-2.5 ± 0.5	PDG 2002 estimate [21]

While the experiments at Mainz and Bates measured at $Q^2 \sim 0.1$ GeV^2 in order to get the $C2/M1$ ratio close to the photon point, the experiments at JLab and Bonn were motivated by the possibility of determining the range of momentum transfers where perturbative QCD (pQCD) would become applicable. In the limit of $Q^2 \to \infty$, pQCD predicts [22] that only helicity-conserving amplitudes contribute, leading to $R_{EM} = E_{1+}^{(3/2)}/M_{1+}^{(3/2)} \to 1$ and $R_{SM} = S_{1+}^{(3/2)}/M_{1+}^{(3/2)} \to$ const.

2 Photo- and electroproduction

According to the Watson theorem, at least up to the two-pion threshold, the ratio $E_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ is a real quantity. However, it is not a constant but even a rather strongly energy-dependent function. If we determine the resonance position as the point, where the phase $\delta_{1+}^{(3/2)}(W = M_\Delta) = 90°$, we can define the so-called "full" ratio

$$R_{EM} = \left.\frac{E_{1+}^{(3/2)}}{M_{1+}^{(3/2)}}\right|_{W=M_\Delta} = \left.\frac{\mathrm{Im}E_{1+}^{(3/2)}}{\mathrm{Im}M_{1+}^{(3/2)}}\right|_{W=M_\Delta}. \qquad (3)$$

We note that this ratio is identical to the ratio obtained with the K-matrix at the K-matrix pole $W = M_\Delta$. This can be seen by using the relation between the T- and the K-matrix, $T = K \cos\delta e^{i\delta}$ and, consequently, $K = \mathrm{Re}T + \mathrm{Im}T \tan\delta$. Therefore, at $W = M_\Delta$ we find

$$K(E_{1+}^{(3/2)})/K(M_{1+}^{(3/2)}) = \mathrm{Im}E_{1+}^{(3/2)}/\mathrm{Im}M_{1+}^{(3/2)}. \qquad (4)$$

The recent, nearly model-independent value of the Mainz group at $W = M_\Delta = 1232$ MeV is $(-2.5 \pm 0.1 \pm 0.2)\%$ [18] in excellent agreement with our dispersion theoretical calculation that gives $(-2.54 \pm 0.10)\%$, see table 1.

As was demonstrated in different approaches [19,23], the precise $E2/M1$ ratio is very sensitive to the specific database used in the fit. Therefore, the SAID value, obtained with the full database is rather low (-1.5% in the previous and -2.0% in the most recent analysis) and the values obtained with the LEGS differential cross-sections are twice as large, around -3%. In order to clearly distinguish the uncertainties arising from different databases from model errors arising from different theoretical approaches used in the partial-wave analysis, a BRAG [24] study group was formed in 2000/2001 to analyze an benchmark dataset of pion photoproduction. Altogether, 7 groups participated in this multipole analysis and determined independently the helicity amplitudes of the Δ excitation and the E/M ratio. The results were reported on the NSTAR2001 workshop in Mainz and showed a surprisingly small spread in R_{EM} of only ± 0.27, which is of about the same size as the systematical uncertainty in the experimental analysis. Details of this study can be found in ref. [25].

For our analysis of pion electroproduction we will use the dynamical model DMT and the unitary isobar model MAID. In the dynamical approach to pion photo- and electroproduction [26], the t-matrix is expressed as

$$t_{\gamma\pi}(E) = v_{\gamma\pi} + v_{\gamma\pi}\, g_0(E)\, t_{\pi N}(E)\,, \qquad (5)$$

where $v_{\gamma\pi}$ is the transition potential operator for $\gamma^* N \to \pi N$, and $t_{\pi N}$ and g_0 denote the πN t-matrix and free propagator, respectively, with $E \equiv W$ the total energy in the CM frame. A multipole decomposition of eq. (5) gives the physical amplitude in channel α [26],

$$t^{(\alpha)}_{\gamma\pi}(q_E, k; E + i\epsilon) = \exp\left(i\delta^{(\alpha)}\right) \cos\delta^{(\alpha)} \times \Bigg[v^{(\alpha)}_{\gamma\pi}(q_E, k) + P\int_0^\infty \mathrm{d}q' \frac{q'^2 R^{(\alpha)}_{\pi N}(q_E, q'; E)\, v^{(\alpha)}_{\gamma\pi}(q', k)}{E - E_{\pi N}(q')} \Bigg]\,, \qquad (6)$$

where $\delta^{(\alpha)}$ and $R^{(\alpha)}_{\pi N}$ are the πN scattering phase shift and reaction matrix in channel α, respectively; q_E is the pion on-shell momentum and $k = |\mathbf{k}|$ is the photon momentum. The multipole amplitude in eq. (6) manifestly satisfies the Watson theorem and shows that the $\gamma\pi$ multipoles depend on the half-off-shell behaviour of the πN interaction.

In a resonant channel like (3,3) in which the $\Delta(1232)$ plays a dominant role, the transition potential $v_{\gamma\pi}$ consists of two terms,

$$v_{\gamma\pi}(E) = v^B_{\gamma\pi} + v^\Delta_{\gamma\pi}(E)\,, \qquad (7)$$

where $v^B_{\gamma\pi}$ is the background transition potential and $v^\Delta_{\gamma\pi}(E)$ corresponds to the contribution of the bare Δ.

It is well known that for a correct description of the resonance contributions we need, first of all, a reliable description of the nonresonant part of the amplitude. In MAID2000, the S, P, D and F waves of the background contributions are complex numbers defined in accordance with the K-matrix approximation,

$$t^{B,\alpha}_{\gamma\pi}(\mathrm{MAID}) = \exp\left(i\delta^{(\alpha)}\right) \cos\delta^{(\alpha)} v^{B,\alpha}_{\gamma\pi}(W, Q^2). \qquad (8)$$

From eqs. (6) and (8), one finds that the difference between the background terms of MAID and of the dynamical model is that off-shell rescattering contributions (principal value integral) are not included in MAID. To take account of the inelastic effects at the higher energies, we replace $\exp(i\delta^{(\alpha)})\cos\delta^{(\alpha)} = \frac{1}{2}(\exp(2i\delta^{(\alpha)}) + 1)$ in eqs. (6) and (8) by $\frac{1}{2}(\eta_\alpha \exp(2i\delta^{(\alpha)}) + 1)$, where η_α is the inelasticity. In our actual calculations, both the πN phase shifts $\delta^{(\alpha)}$ and inelasticity parameters η_α are taken from the analysis of the GWU group [27].

Following ref. [28], we assume a Breit-Wigner form for the resonance contribution $\mathcal{A}^R_\alpha(W, Q^2)$ to the total multipole amplitude,

$$\mathcal{A}^R_\alpha(W, Q^2) = \bar{\mathcal{A}}^R_\alpha(Q^2)\, \frac{f_{\gamma R}(W)\Gamma_R\, M_R\, f_{\pi R}(W)}{M_R^2 - W^2 - iM_R\Gamma_R}\, e^{i\phi}, \qquad (9)$$

where $f_{\pi R}$ is the usual Breit-Wigner factor describing the decay of a resonance R with total width $\Gamma_R(W)$ and physical mass M_R. The expressions for $f_{\gamma R}$, $f_{\pi R}$ and Γ_R are given in ref. [28]. The phase $\phi(W)$ in eq. (9) is introduced to adjust the phase of the total multipole to equal the corresponding πN phase shift $\delta^{(\alpha)}$. Because $\phi = 0$ at resonance, $W = M_R$, this phase does not affect the Q^2-dependence of the $\gamma N R$ vertex.

The resonance couplings for $\alpha = M, E, S$ are parameterized in the following way:

$$\bar{\mathcal{A}}^\Delta_\alpha(Q^2) = X^\Delta_\alpha(Q^2)\, \bar{\mathcal{A}}^\Delta_\alpha(0) \frac{k}{k_W}\, F(Q^2)\,,$$
$$F(Q^2) = (1 + \beta\, Q^2)\, e^{-\gamma Q^2}\, G_D(Q^2)\,, \qquad (10)$$

where $k = (Q^2 + (W^2 - m^2 - Q^2)^2/(4W^2))^{1/2}$ is the virtual photon momentum, $k_W = (W^2 - m^2)/(2W)$ is the equivalent photon momentum or energy and $G_D(Q^2) = 1/(1 + Q^2/0.71)^2$ is the usual dipole form factor. The parameters β and γ were determined by fitting $\bar{\mathcal{A}}^\Delta_M(Q^2)$ to the data for G^*_M [28,29]. In the case of MAID we obtained $\beta = 0$ and $\gamma = 0.21$ (GeV/c)$^{-2}$. In MAID2000 the coefficients $X^\Delta_\alpha(Q^2)$ are kept constant but can be used as free parameters in single-Q^2 fits. The values of $\bar{\mathcal{A}}^\Delta_M(0)$ and $\bar{\mathcal{A}}^\Delta_E(0)$ were determined by fitting to the multipoles obtained in the recent analyses of the Mainz [19] and GWU [20] groups.

With the definition of eq. (9), the background amplitudes of the P_{33} channel vanish exactly at the resonance position and the resonance amplitudes become purely imaginary. In this case the helicity amplitudes $A_{1/2}, A_{3/2}, S_{1/2}$, which are the characteristic numbers for e.m. resonance excitation, are directly related to the resonance multipoles at $W = M_\Delta$ (see, *e.g.*, PDG94 or ref. [30]),

$$A_{1/2} = -\frac{1}{\sqrt{6}} a_\Delta (\bar{M}^{(3/2)}_{1+} + 3\bar{E}^{(3/2)}_{1+})\,,$$
$$A_{3/2} = -\frac{1}{\sqrt{2}} a_\Delta (\bar{M}^{(3/2)}_{1+} - \bar{E}^{(3/2)}_{1+})\,,$$
$$S_{1/2} = -\frac{2}{\sqrt{3}} a_\Delta \bar{S}^{(3/2)}_{1+}\,,$$
$$\text{with } a_\Delta = \left(\frac{4\pi q_\Delta M_\Delta \Gamma_\Delta}{k_W m}\right)^{1/2}. \qquad (11)$$

Here we use the notation $\bar{M}^{(3/2)}_{1+} = \mathrm{Im} M^{(3/2)}_{1+}(W = M_\Delta)$, etc. In MAID2000 these are identical to $\bar{\mathcal{A}}^\Delta_\alpha(Q^2)$. Using this definition we can now define the $N \to \Delta$ transition form factors,

$$
\begin{aligned}
G_M^*(Q^2) &= \ \ b_\Delta \bar{M}_{1+}^{(3/2)}(Q^2) = b_\Delta \bar{\mathcal{A}}_M^\Delta(Q^2)\,,\\
G_E^*(Q^2) &= -b_\Delta \bar{E}_{1+}^{(3/2)}(Q^2) = -b_\Delta \bar{\mathcal{A}}_E^\Delta(Q^2)\,,\\
G_C^*(Q^2) &= -b_\Delta \bar{S}_{1+}^{(3/2)}(Q^2) = -b_\Delta \bar{\mathcal{A}}_C^\Delta(Q^2)\,,\\
\text{with } b_\Delta &= \left(\frac{8m^2 q_\Delta \Gamma_\Delta}{3\alpha_{\rm em} k_\Delta^2}\right)^{1/2}
\end{aligned} \tag{12}
$$

and $\alpha_{\rm em} = 1/137$, $\Gamma_\Delta = 115$ MeV. k_Δ and q_Δ are the photon and pion momenta at $W = M_\Delta$.

The e.m. transition form factors may also be expressed in terms of the helicity amplitudes $A_{1/2}, A_{3/2}$ and $S_{1/2}$, which are determined at the resonance position $W = M_\Delta$ and are functions of Q^2,

$$
\begin{aligned}
G_M^* &= -c_\Delta (A_{1/2} + \sqrt{3} A_{3/2})\,,\\
G_E^* &= \ \ c_\Delta \left(A_{1/2} - \frac{1}{\sqrt{3}} A_{3/2}\right)\,,\\
G_C^* &= \sqrt{2} c_\Delta S_{1/2}\,,\\
\text{with } c_\Delta &= \left(\frac{m^3 k_W}{4\pi\alpha M_\Delta k_\Delta^2}\right)^{1/2}.
\end{aligned} \tag{13}
$$

The E/M and S/M ratios can then be defined as

$$
\begin{aligned}
R_{EM} &= -\frac{G_E^*}{G_M^*} = \frac{A_{1/2} - \frac{1}{\sqrt{3}} A_{3/2}}{A_{1/2} + \sqrt{3} A_{3/2}}\,,\\
R_{SM} &= -\frac{G_C^*}{G_M^*} = \frac{\sqrt{2} S_{1/2}}{A_{1/2} + \sqrt{3} A_{3/2}}\,.
\end{aligned} \tag{14}
$$

At this point we note that there is not yet a commonly accepted definition of the G_C^* form factor in the literature. Here we choose a definition which leads to a relation for the S/M ratio similar to the E/M ratio and which leads to a similar behaviour for low Q^2. However, other conventions exhibit different normalizations at $Q^2 = 0$.

Furthermore, the helicity asymmetry A_1^Δ can be expanded for small E/M ratios,

$$
A_1^\Delta = \frac{A_{1/2}^2 - A_{3/2}^2}{A_{1/2}^2 + A_{3/2}^2} \approx -\frac{1}{2} + 3R_{EM} + O(R_{EM}^2) \tag{15}
$$

which is practically constant and varies only between -0.58 at $Q^2 = 0$ and -0.50 at $Q^2 \approx 4$ GeV^2. This is in big contrast to the next important resonances of pion electroproduction, the $D_{13}(1520)$ and the $F_{15}(1680)$, which show a very rapid cross-over from -1 at $Q^2 = 0$ to $+1$ already for $Q^2 \approx 2$ GeV^2.

At $Q^2 = 0$ the form factors can be related to transition moments, a magnetic-dipole moment and an electric-quadrupole moment. Furthermore, as the resonance production is far away from the Siegert limit ($\omega = 258.7$ MeV), the quadrupole moments from $E2$ and $C2$ transitions can be different. First we express the form factors in the spherical notation G_{M1}, G_{E2}, G_{C2}, which lead to the standard definition of transition moments in the limit of $Q^2 = 0$,

$$
\begin{aligned}
G_M^*(0) &= \sqrt{\frac{2m}{3M_\Delta}} G_{M1}(0) = \sqrt{\frac{m}{M_\Delta}} \mu_{N\Delta}\,,\\
G_E^*(0) &= \sqrt{\frac{2m}{3M_\Delta}} G_{E2}(0) = -\frac{m k_W}{6} \sqrt{\frac{m}{M_\Delta}} Q_{N\Delta}\,.
\end{aligned} \tag{16}
$$

With eqs. (14),(16) the quadrupole transition moment can then be expressed in terms of the magnetic transition moment and the E/M ratio at the real photon point and

Table 2. Recent experimental data of π^0 electroproduction on the proton. The Mainz experiment was done with beam and recoil polarization, all others are unpolarized measurements.

Laboratory	Q^2 (GeV^2)	$W_{\rm cm}$ (MeV)	$\theta_\pi^{\rm cm}$ (degrees)
Mainz [31]	0.121	1232	180
Bates [32]	0.126	1152–1322	0–38
Bonn [33]	0.630	1153–1312	5–175
JLab, Hall A [34]	1.0	1110–1950	146–167
JLab, Hall B [35]	0.4–1.8	1100–1680	26–154
JLab, Hall C [36]	2.8, 4.0	1115–1385	25–155

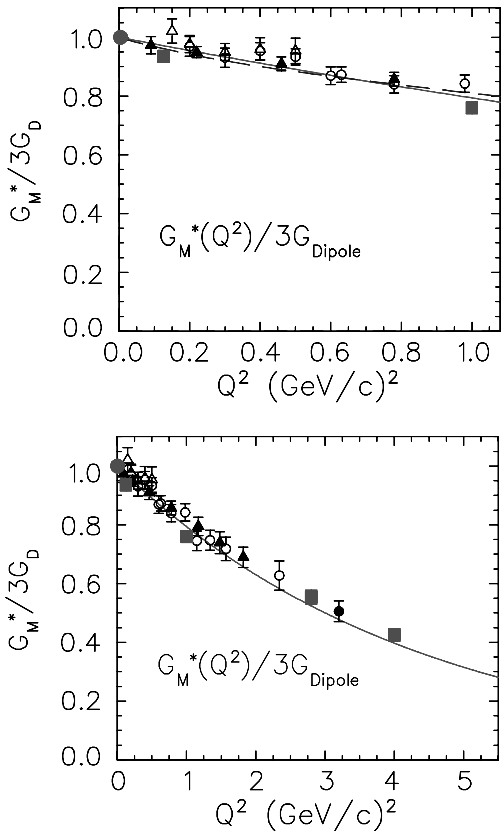

Fig. 1. The Q^2-dependence of the magnetic $N \to \Delta$ transition form factor G_M^* divided by three times the nucleon dipole form factor. The solid and dashed curves are the results of the MAID and dynamical model analyses, respectively. The data at $Q^2 =$ 1.0, 2.8 and 4.0 (GeV/c)^2 are from our own analysis. For other data see ref. [37].

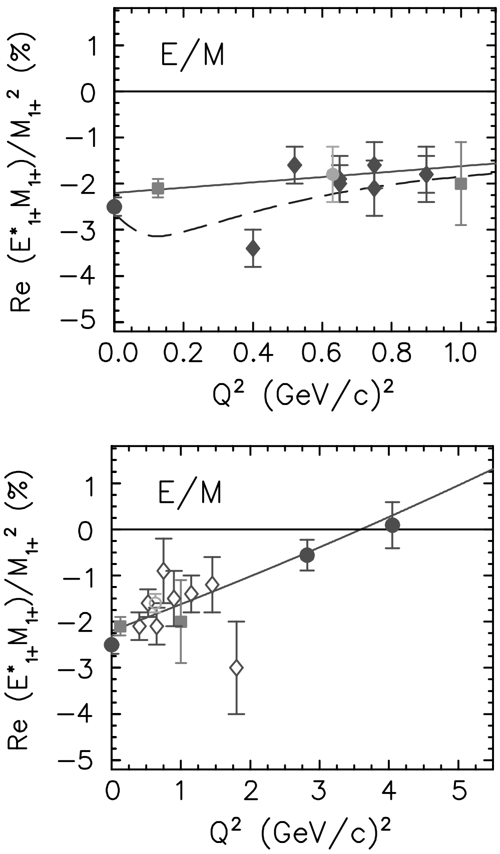

Fig. 2. The Q^2-dependence of the E/M ratio R_{EM} at $W = 1232$ MeV. The solid and dashed curves are the MAID and dynamical model results, respectively. Experimental data at $Q^2 = 0$ from ref. [17]. In the upper panel, the point at $Q^2 = 1.0$ is from our analysis to the JLab Hall A data [34], the point at $Q^2 = 0.1$ is from Bates [32], the circle at $Q^2 = 0.63$ from Bonn [33] and the points at $Q^2 = 0.4 \cdots 0.9$ from JLab Hall B [35]. In the lower panel all points show our own single-Q^2 analysis.

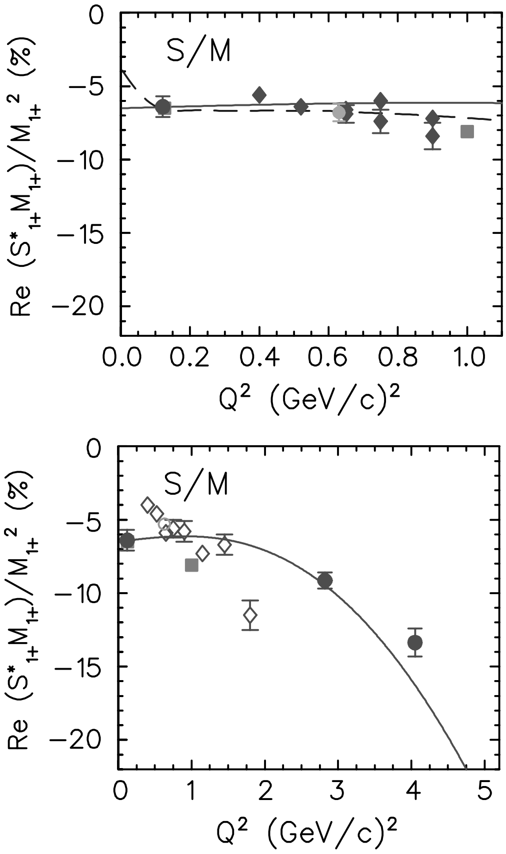

Fig. 3. The Q^2-dependence of the S/M ratio R_{SM} at $W = 1232$ MeV. The points at $Q^2 = 0.1$ are from Mainz [31] (circle) and Bates [32] (square). Other notations are the same as in fig. 2.

evaluated from the Mainz multipole analysis,

$$\begin{aligned} Q_{N\Delta} &= \frac{6\mu_{N\Delta}}{mk_W} R_{EM}(0) \\ &= -(0.0846 \pm 0.0033)\,\mathrm{fm}^2 . \end{aligned} \tag{17}$$

3 Data analysis

The unitary isobar model MAID was used to analyze recent differential cross-section data on $p(e,e'p)\pi^0$ from Mainz, Bates, Bonn and JLab. These data cover a Q^2 range from 0.1 to 4.0 $(\mathrm{GeV}/c)^2$ and an energy range $1.1 < W < 2.0$ GeV, see table 2. In a first attempt we have fitted each data set at a constant Q^2 value separately. This is similar to a partial-wave analysis of pion photoproduction and only requires additional longitudinal couplings for all the resonances. The Q^2 evolution of the background, Born terms and vector meson exchange, is described with a standard dipole form factor. Our results for the G_M^* form factor are shown in fig. 1.

It is worth noting that in the definition of eq. (17), $G_M^*(0)/3$ takes a value of 1 to an accuracy of 1%. This very precise value is extracted from the recent Mainz experiment [18]. With this number we can also determine a very precise $N \to \Delta$ magnetic transition moment, $\mu_{N\Delta} = 3.46 \pm 0.03$ in units of nuclear magnetons.

Our extracted values for R_{EM} and R_{SM} and a comparison with the results of refs. [33,35,36] are shown in figs. 2 and 3, respectively. The main difference between our results and those of ref. [36] is that our values of R_{EM} show a clear tendency to cross zero and change sign as Q^2 increases. This is in contrast with the results obtained in the original analysis [36] of the data which concluded that R_{EM} would stay negative and tend toward more negative values with increasing Q^2.

In addition to our single-Q^2 fits, we have also performed a global fit by parameterizing the transition form factors with simple Q^2-dependent functions. Most suitable we found the following ansatz:

$$\begin{aligned} X_M(Q^2) &= (1 + \alpha_M Q^2)e^{-\beta_M Q^2} , \\ X_E(Q^2) &= (1 + \alpha_E Q^2)e^{-\beta_E Q^2} , \\ X_C(Q^2) &= (1 + \alpha_C Q^6)e^{-\beta_C Q^2} . \end{aligned} \tag{18}$$

Table 3. Result of our Q^2-dependent fit to the data of Bonn, and JLab Hall A, B, C.

	$M1$	$E2$	$C2$
α	-0.0049 ± 0.009	-0.278 ± 0.035	0.0410 ± 0.007
β	0.016 ± 0.008	0	0.119 ± 0.025

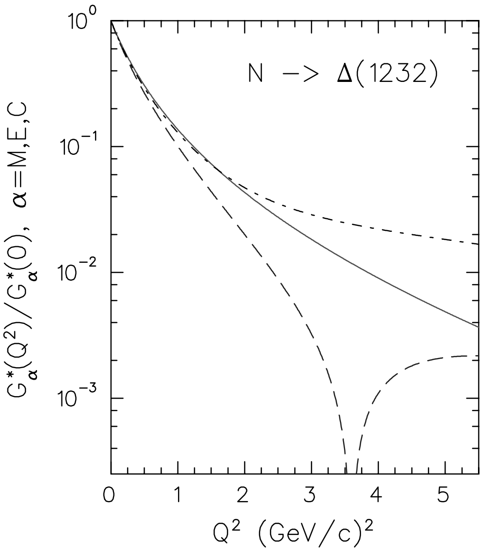

Fig. 4. The electromagnetic-transition form factors of the $\Delta(1232)$ excitation in our parametrization of eqs. (10),(12),(18) as a result of the fit to the electroproduction data. The solid, dashed and dash-dotted lines show the G_M^*, G_E^* and G_C^* form factors, respectively.

At $Q^2 = 0$ we have fixed the multipoles to the values determined in our photoproduction analysis and fitted the 6 parameters α_i, β_i of the Q^2 evolution to the electroproduction data of Bonn and JLab Hall A, B, C. The result of our fit (table 3) is shown in figs. 1-4. Similarly to our single-Q^2 fit we find a zero-crossing of the electric form factor at the position $Q^2 = 3.6 \pm 0.5$ (GeV/c)^2.

4 Conclusions

At the resonance position, where the phase passes 90°, we obtain an $E2/M1$ ratio of $R_{EM} = (-2.5 \pm 0.1)\%$.

For pion electroproduction, we have analyzed recent Bonn and JLab data for electroproduction of the $\Delta(1232)$-resonance via $p(e, e'p)\pi^0$ with our unitary isobar model MAID, which gives an excellent description of the existing database. In contrast to previous findings, our model indicates that R_{EM}, starting from a small and negative value at the real photon point, actually exhibits a clear tendency to cross zero and changes sign as Q^2 increases. It will be most interesting to have data at yet higher momentum transfer in order to see whether such a trend continues, which would be a sign for a rather slow approach towards the pQCD region. Furthermore, the absolute value of R_{SM} is strongly increasing, which indicates that the pQCD prediction of $R_{SM} \rightarrow$ constant is not yet reached.

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